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Rolling friction of a rugby wheelchair

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Abstract

The rolling friction coefficient is treated either as a constant, or a constant plus a parabolic velocity-dependent component. According to mathematical models, the rolling friction coefficient of visco-elastic objects increases first and drops at higher speeds. The aim of this paper was to analyse the non-linear rolling friction of a rugby wheelchair as a function of speed. Non-linear velocity-dependent drag and lift coefficients were determined in the wind tunnel. In order to obtain the friction coefficient, we applied the coast down method on three different floors (wood, linoleum, and short-pile carpet) by instrumenting the wheelchair with an accelerometer. The rolling friction coefficient was calculated from the ratio of the difference between inertial force and drag force to the difference between weight and upward lift force (where all forces are absolute values). The friction coefficient μ of the carpet floor was the highest ($\mu = 0.0143$; mean weighted to velocity), followed by the linoleum floor ($\mu = 0.0061$) and the wooden one ($\mu = 0.0042$). μ of carpet and linoleum showed no clear trend in the velocity range of 0-4 m s^{-1} and can be treated as a constant. μ of the wooden gym floor increased, then dropped and increased again with speed. A fit function based on a combined Bateman and parabolic function was applied to the wood data in order to separate the initial peak from the velocity dependency at higher speeds.

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1. Introduction

At low speeds, rolling resistance is the major source of energy loss in balls and vehicles, depending on the rolling friction coefficient of balls and tyres on different surfaces, and also on the mass of the vehicle. Several methods for measuring and analysing rolling friction are available, e.g.

- coast down methods:
  - - distance of roll [1-3] by applying ramp methods,
  - - timing gate method, which measures the velocity in discrete steps between the gates, and estimates the velocity decreasing with time [4,5],
  - - velocity method, which directly measures the velocity with time; e.g. [6,7];
- conveyor belt method, which uses large diameter rotating drums and flat conveyor belts to directly measure the rolling resistance [8],
- pulling method, where a rolling object is pulled on a flat level surface, the reaction force is measured and the rolling resistance calculated based on the mass of the object, and
- oscillation method, where an eccentric mass is attached in between two tyres which are allowed to rock back and forth until they come to a stop; the rate of angular decay represents the friction resistance [9,10].

The disadvantage of all these methods is that the coefficient of rolling friction is considered constant and thus velocity independent:

$$F_R = \mu_R mg \tag{1}$$

where F_R is the friction force, μ_R is the coefficient of rolling friction, and the product of mass and gravitational acceleration, $m\,g$, is the weight of the object.

Petrushov [11] introduced a coast down method which allows extracting the velocity dependent rolling friction as a parabolic function of the velocity v

$$F_R = \mu_R mg + k_f mgv^2 \tag{2}$$

where k_f is the coefficient of speed influence on the rolling resistance [11]. In the case of car tyres, $\mu_R = 0.009 - 0.014$, and $k_f = 1\text{x}10^{-6} - 1\text{x}10^{-5}\ \text{s}^2\ \text{m}^{-2}$ [11], which are functions of tyre inflation pressure and temperature (data from indoor vehicle coast-down tests on a wooden floor).

During coasting down, the inertial force, rolling resistance and aerodynamic drag force are in equilibrium, resulting in the following non-linear ordinary differential equation

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + 0.5\rho c_D A v^2 + k_f mgv^2 + \mu_R mg = 0 \tag{3}$$

or

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + c_1 v^2 + c_2 = 0 \tag{4}$$

where ρ is the density of air, c_D is the drag coefficient, A is the frontal area, and c_1 and c_2 combine the constants of Eq. (3). Solving Eq. (4) for v yields

$$v_t = \sqrt{\frac{c_2}{c_1}}\tan\left[\tan^{-1}\left(v_0\sqrt{\frac{c_1}{c_2}}\right) - t\frac{\sqrt{c_1 c_2}}{m}\right] \tag{5}$$

where v_t is the instantaneous velocity decreasing with time when coasting-down, and v_0 is the initial condition at impending deceleration [11]. The constant c_2 combines the lumped drag coefficient, $0.5\rho c_D A$, and the velocity dependent rolling friction coefficient times the weight, $k_f mg$. Separating both values is only possible if the lumped drag coefficient is known from wind tunnel experiments.

Mathematical analyses of spheres or cylinders rolling on a flat surface, where one object is visco-elastic and the other rigid, show that rolling resistance increases initially with speed before decreasing at higher speeds [12,13]. This behaviour would add another component to the velocity dependent rolling friction coefficient k_f known from car tyres [11].

Wheelchair rugby was developed for quadriplegic athletes as an alternative to wheelchair basketball. It is a combination of both rugby and basketball and the main aim of the game is to carry the ball across the opponent's goal line, and likewise to prevent the other team from getting the ball or scoring. Like the game of rugby, wheelchair rugby requires the athletes to turn and accelerate quickly to avoid their opponents and score. Therefore a rugby wheelchair needs to be highly manoeuvrable and be able to accelerate quickly from standstill. The ability to accelerate a rugby wheelchair from standstill is determined by three major characteristics: a) those related to wheelchair configuration such as the overall rolling resistance and internal friction; b) those related to the athlete such as explosive strength and propulsion technique and c) those related to the adjustment of the wheelchair characteristics to the functional abilities of the player or wheelchair-user interface [14]. Rugby wheelchairs (as well as most designs of basketball and some tennis chairs) are six-wheeled in contrast to other wheelchair types. Two of the six wheels are large diameter inflatable spoke or disc tyres, and the other four are small size plastic casters with either the front or the rear pair in contact with the ground.

Hoffman *et al.* [6] determined the rolling friction coefficients of several wheelchair types (folding, rigid ultra light and racing) on linoleum and carpet. Their friction coefficient data range from 0.0150-0.0212 on carpet and from 0.0013-0.0099 on linoleum. Fuss [7] investigated the rolling friction coefficients in racing wheelchairs on rubber tracks which amounted to 0.01-0.012.

The aim of this study was to analyse the non-linearity of the rolling friction coefficient of a rugby wheelchair, by

- determining its non-linear lumped drag and lift coefficients as functions of speed,
- determining the coast-down deceleration and speed on three different surfaces, and
- presenting the rolling friction coefficient as a function of speed.

2. Experimental

1) Rugby wheelchair: all experiments were carried out with a rugby wheelchair custom built by Melrose (Christchurch, New Zealand; Figure 1). The weight of the chair was 169 N.

2) Wind tunnel experiments: drag and lift were measured using a Type 9260AA6 Kistler force plate (Kistler, Winterthur, Switzerland). The force plate was mounted in an industrial wind tunnel, the test section of which is 3 m wide, 2 m high and 9 m long. The maximum attainable wind speed is up to 40 m s^{-1}, but required test speed for this experiment was only up to 11.11 m s^{-1} (40 kph). As the support area of the rugby wheelchair is larger than the force plate, a wooden board (1.8 x 0.85 x 0.03 m) was secured onto the force plate before the wheelchair was placed on top (Figure 1). The drag and lift force was obtained from the force plate plus the wooden board first, at increments of 5 kph. Subsequently, the wheelchair was placed on the board and the procedure was repeated. Drag and lift forces were recorded using the Kistler Bioware (Kistler, Winterthur, Switzerland) software. The drag and lift forces of the force plate plus board were subtracted from the wheelchair drag and lift forces, and divided by the velocity squared. A reciprocal function of the structure

$$C_D, C_L = \frac{A}{v+B} + D \tag{6}$$

where C_D and C_L are the lumped drag and lift coefficients ($0.5\rho c_D A$, $0.5\rho c_L A$), respectively, and A, B and D are the parameters of this function (D is the asymptotic value), was fitted to the lumped drag and lift coefficients to account for the increased drag and lift at small velocities.

3) Coast down experiments: the experiments were carried out on three different indoor surfaces, a short pile carpet floor, a linoleum floor, and a standard area-elastic wooden gym floor. The axles of the small front wheels were aligned such that their vectors are parallel with zero distance in between, and the wheels were locked in this

position. This served to ensure a straight cost down path and avoid caster shimmy. The wheelchair was accelerated manually and allowed to coast down until standstill or manual deceleration. The latter enabled higher coast-down speeds. The experiments were carried out at different initial speeds (“high”, “medium”, “slow”) and repeated three times. For “slow” speeds, the wheelchair reached a standstill, and for higher velocities it was decelerated manually. Overlapping of the velocity ranges of “high”, “medium”, “slow” speeds at free coast down (without manual deceleration) was essential and thus aimed at. The acceleration data were collected with an accelerometer (minimaxX, Catapult Innovations Pty Ltd, Scoresby, Australia; resolution 0.037 m s^{-2}), attached to the frame of the wheelchair, at 100 Hz.

3. Data analysis

The acceleration a was numerically integrated with time in order to obtain the velocity. The slight offset of the data at zero acceleration was corrected such that the velocity after integration equals zero when the wheelchair is not moving. As the rolling friction coefficient μ is the ratio of tangential friction force to weight, μ was obtained from

$$\mu = -\frac{F_I + F_D}{F_G + F_L} \tag{7}$$

where F_I, F_D, F_G and F_L are inertial, drag, gravitational and lift forces, respectively, or

$$\mu = -\frac{-am + C_D v^2}{gm + C_L v^2} \tag{8}$$

Note that inertial and drag forces are opposite, as are gravitational and lift forces. F_G and F_D are negative in Eqs. (7) and (8).

The rolling friction coefficient μ was expressed as a function of velocity. The noise was removed with a 2nd order Savitzky-Golay filter of a window width of 1/33 of all data, applied five times.

As the non-linear rolling friction at slow velocities (c.f. Figure 3 of [12], and Figure 4 of [13]) resembles a Bateman function, a combined Bateman and parabolic function of the form

$$\mu = A\left(\mathrm{e}^{-Bv} - \mathrm{e}^{-Cv}\right) + Dv^2 \tag{9}$$

was fitted to the friction coefficients of the three different surfaces, where A, B, C, and D are the coefficients of this function ($D = k_f mg$).

4. Results

The lumped drag and lift coefficients, C_D and C_L, against velocity v are shown in Figure 2. The corresponding equations are:

$$C_D = \frac{0.2955}{v + 0.0762} + 0.1608 \tag{10}$$

and

$$C_L = \frac{0.2099}{v + 0.1218} + 0.1666 \tag{11}$$

The rolling friction coefficient μ against velocity v are shown in Figure 3. μ of the carpet floor was the highest (μ = 0.0143; mean weighted to velocity), followed by the linoleum floor (μ = 0.0061) and the wooden one (μ =

0.0042). The fluctuations of μ in Figure 3 indicate a range where the actual μ is located, and should not be interpreted e.g. that μ of the carpet floor at v = 1.9 m s^{-1} is exactly 0.01567. The combined Bateman and parabolic function of Eq. (9) was fitted in the data of the wooden floor only; the data of the other floors did not return all positive coefficients. Only the data of the wooden floor showed a clear initial increase in μ, followed by a decrease and another increase. The corresponding fit function (Figure 3) is

$$\mu = 0.00808\left(\mathrm{e}^{-0.8707v} - \mathrm{e}^{-28.45v}\right) + 0.00051v^2 \tag{12}$$

As $k_f mg$ equals 0.00051 in Eq. (12), k_f amounts to 3x10^{-6} s^2 m^{-2}.

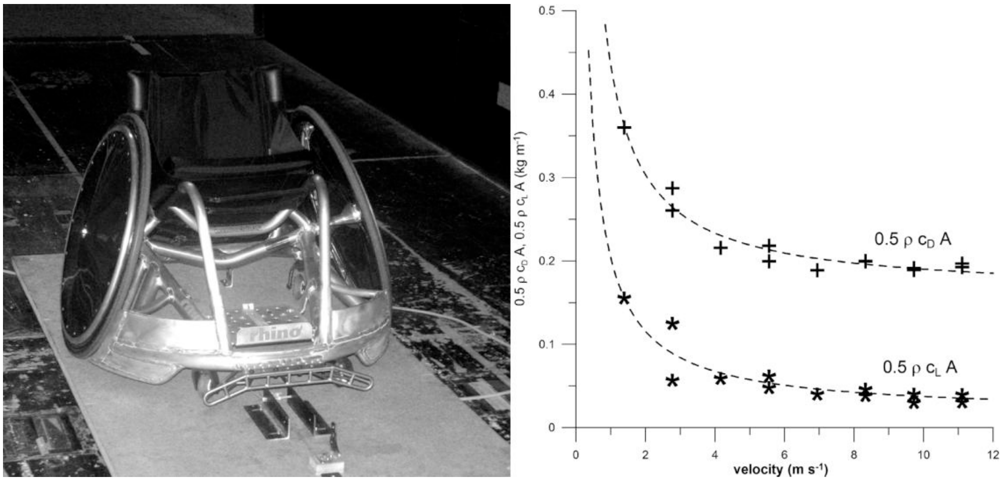

Fig.1. (a) Rugby wheelchair mounted on the force plate in the wind tunnel; (b) lumped drag and lift coefficients against velocity; dashed lines: curve fit according to Eq. (6).

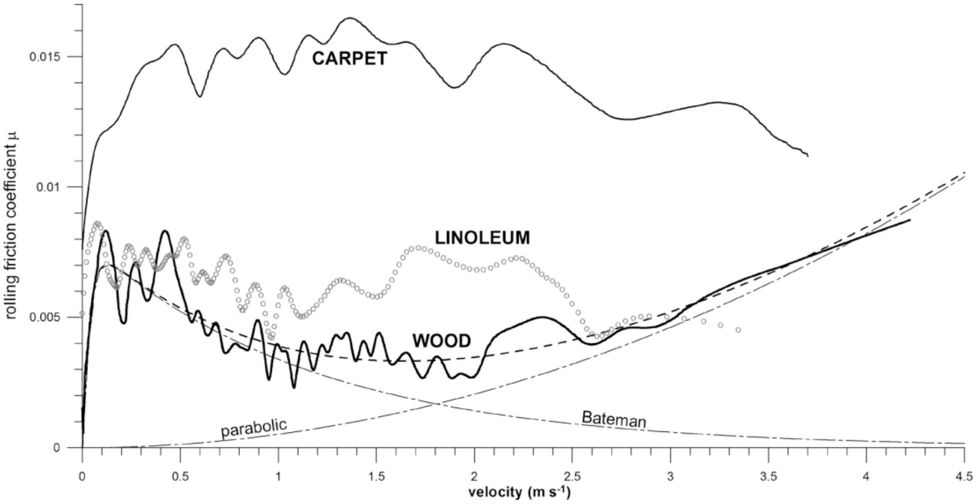

Fig. 2. Rolling friction coefficient against velocity; dashed lines: curve fit according to Eq. (9).

5. Discussion

The method used for determining the non-linear friction coefficient μ as a function of velocity, applied in this study, was successful. Expressing μ non-linearly, however, makes sense only if the data follow roughly a combined Bateman and parabolic function. This was not the case in the data of carpet and linoleum floors and thus a single, constant μ is recommended. k_f of the wheelchair on the wooden floor, which was $3\text{x}10^{-6}$ s^2 m^{-2} in this study, lies well within the range of Petrushov's results, which is between $1\text{x}10^{-6}$ and $1\text{x}10^{-5}$ s^2 m^{-2} [11].

The initial peak of μ occurs at v = 0.1264 m s^{-1}. Considering that the main wheels have a diameter of 0.61 m a linear speed of 0.1264 m s^{-1} corresponds to an angular one of 0.4144 rad s^{-1}. According to [12] and [13], the velocity at the peak μ depends on the radius and weight of the rolling object, and on the elastic, viscous and inertial parameters as well as the Poisson ratio of the visco-elastic object (rolling or surface).

The data of Hoffman *et al.* [6] indicate that the drag area (c_DA) of the same wheelchair is larger on carpet than on linoleum. As this is impossible, their result suggests that k_f on carpet is larger than k_f on linoleum and that both surfaces have a velocity dependent component. Hoffman *et al.* [6] did not consider that c_1 in Eq. (4) is not just $0.5\rho c_DA$ but rather $0.5\rho c_DA + k_fmg$. The carpet data of Hoffman *et al.* (μ = 0.0150-0.0212) are comparable to ours (μ = 0.0143). Their linoleum data range from 0.0013-0.0099. It has to be noted that Hoffman *et al.* [6] did not investigate rugby wheelchairs, which are of a different construction (two large wheels, two plastic casters in contact with the ground) than their folding, rigid ultra light and racing wheelchairs.

References

[1] Van der Plas R. Rolling resistance of bicycle tires. *Bike Tech* 1983;**2**:8-12.
[2] Fehlau G. *The recumbent bicycle*. Ann Arbor MI: Out Your Backdoor Press; 2000.
[3] FIFA Quality Concept for Football Turf. Available: http://www.fifa.com/mm/document/afdeveloping/pitchequip/fqc_football_turf_folder_342.pdf.
[4] British Standard BS7044. Artificial sports surfaces, Part 2 Section 2.1. London: British Standards Institution; 1991.
[5] Kolitzus HJ. Ball Roll Behavior: The Functional Relationship of the Ball Roll Distance and the Timing Gate Method; How to Calculate the Ball Roll Distance from Timing Gate Measurements. ISSS - International Association for Sports Surface Sciences, Eschenz / Switzerland, 2003. Available: http://www.isss-sportsurfacescience.org/downloads/documents/ZPKPAJJUWY_Ball_Roll_BehaviorKS.pdf.
[6] Hoffman MD, Millet GY, Hoch AZ, Candau RB. Assessment of wheelchair drag resistance using a coasting deceleration technique. *Am J Phys Med Rehabil* 2003;**82**:880-889.
[7] Fuss FK. Influence of mass on the speed of wheelchair racing. *Sports Engineering* 2009;**12**:41–53.
[8] Milliken WF, Milliken DF. *Race Car Vehicle Dynamics*. Warrendale PA: SAE International; 1995.
[9] Hill B. Measurement of rolling resistance using an eccentrically weighted oscillating wheel. In: Meyer WE, and Reichert J, editors. *Surface characteristics of roadways*. Philadelphia PA: ASTM; 1990, p. 497-504.
[10] Wang E, Macedo V, Reid J. A method for quantifying the rolling resistance of bicycle tires. In: Hubbard M, Mehta RD, Pallis JM, editors. *The Engineering of Sport 5*, Vol. 2. Sheffield: International Sports Engineering Assocation; 2004, p. 132-138.
[11] Petrushov VA. Improvement in vehicle aerodynamic drag and rolling resistance determination from coast-down tests. *Proc Instn Mech Engrs Part D, J Automobile Engng* 1998;**212**:369-380.
[12] Pöschel T, Schwager T, Brilliantov NV. Rolling friction of a hard cylinder on a viscous plane. *Eur Phys J* 1999;**B10**:169-174.
[13] Yung KL, Xu Y. Non-linear expressions for rolling friction of a soft ball on a hard plane. *Nonlin Dyn* 2003;**33**:33-41.
[14] Vanlandewijck Y, Theisen D, Thaly D. Wheelchair Propulsion Biomechanics. *Sports Med* 2001;**31**: 339-367.